General Algebra Practice

1. (a) If $(ab^3)^{\frac{2}{3}} = (a^3b)^{\frac{1}{3}}$, express a in terms of b in the form $a = b^n$.

   (b) If $y^{\frac{2}{3}} = z$, express $y^{\frac{4}{3}}$ in terms of z, and hence, or otherwise, solve the equation $4y^{\frac{4}{3}} - 37y^{\frac{2}{3}} + 9 = 0$ to find y.

2. (a) Solve the equations:

$$2^{2x+2} - 9 \times 2^x + 2 = 0,$$

   (b) Simplify

$$\frac{4x^{\frac{3}{2}} - x^{-\frac{1}{2}}}{2x^{\frac{1}{2}} - x^{-\frac{1}{2}}}$$

3. Simplify

   (a) $\dfrac{4}{\sqrt{2}} - \dfrac{4}{\sqrt{8}}$,

   (b) $\dfrac{1}{\sqrt{2}}\left(2\sqrt{2} - 1\right) + \sqrt{2}\left(1 - \sqrt{8}\right)$,

   (c) $\dfrac{10}{\sqrt{5}} + \sqrt{20}$,

   (d) $\dfrac{\sqrt{6}}{\sqrt{2}} + \dfrac{3}{\sqrt{3}} + \dfrac{\sqrt{15}}{\sqrt{5}} + \dfrac{\sqrt{18}}{\sqrt{6}}$.

4. Solve the simultaneous equations

$$\begin{aligned} x - 2y &= 3 \\ 2x^2 - 3xy &= 35 \end{aligned}$$

5. (a) If $y = x + \dfrac{1}{x}$ verify that $x^2 + \dfrac{1}{x^2} = y^2 - 2$. Hence find the possible values of x if

$$2\left(x^2 + \frac{1}{x^2}\right) - 16\left(x + \frac{1}{x}\right) + 26 = 0$$

   (b) If $\dfrac{a}{x} = \dfrac{x}{y} = \dfrac{y}{b}$ express x in terms of a and b only, using fractional indices.

6. (a) Multiply out $(x - a)\left(x - \dfrac{1}{a}\right)$ and deduce the two factors of

   $x^2 - \left(\sqrt{2} + \dfrac{1}{\sqrt{2}}\right)x + 1$.

   Solve the equation $x^4 - \dfrac{5x^2}{2} + 1 = 0$.

(b) Simplify

i. $\dfrac{(2x)^{\frac{1}{2}} \times (4x)^{\frac{3}{4}}}{\sqrt{x^{\frac{1}{2}}}}$

ii. $\dfrac{\sqrt{a^3b - ab^3}}{\sqrt{ab(a+b)}}$

7. Solve each of the following inequalities.

(a) $\dfrac{x}{x-2} < 5$

(b) $x(x-2) < 5$

8. (a) Express $9x^2 + 12x + 7$ in the form $(ax+b)^2 + c$ where a, b, c are constants whose values are to be found.

(b) Find the set of values taken by $\dfrac{1}{9x^2 + 12x + 7}$ for real values of x.

9. Solve the equations:

(a) $\sqrt{x} = 16$,

(b) $4x^2 + \dfrac{75}{x^2} = 103$,

(c) $5 \times (5^x)^2 = 126 \times (5^x) - 25$.

10. Find constants a, b and c such that, for all values of x,

$$3x^2 - 5x + 1 = a(x+b)^2 + c.$$

Hence find the coordinates of the minimum point on the graph of $y = 3x^2 - 5x + 1$.

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